Dynamic Monitoring vertical Deflectionof Small Concrete Bridge Using Conventional Sensors And 100 Hz GPS Receivers -Preliminary Results

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Abstract: - The last four decades have been significant for Brazil's highway network development. The country received financial incentives for its expansion and created various structural solutions for bridges and overpasses. Concurrent to this development, in recent years these structures have increasingly shown advanced deterioration stages due to the lack of preventive maintenance programs. Thus, this paper proposes the use of a GPS in a short-term monitoring plan to oversee the structural behavior of a curved reinforced concrete highway bridge already in service. Accordingly, this article presents the first research results with the L1 carrier and recorded data at 100 Hz in order to monitor the dynamic behavior of the central span of a small curved concrete bridge, the bridge over the Jaguari River in Extrema, Minas Gerais. The problem is that such structures exhibitsmall peak-to-peak amplitude vertical deflections- up to 5 mm. The challenge lies in the fact that the vast majority of Brazil's highway network is characterized by small and medium sized concrete bridges. The bridge monitored in the study consists of two traffic lanes and a total length of 134 m, located in the Fernão Dias highway- BR 381, km 946 + 300m, Extrema, Minas Gerais. The bridge under study is significant to this work because this type of structure is largely found throughout the country. Thus, the line of work to be presented will contribute to implementing other monitoring actions for these types of structures throughout Brazil – quickly and effectively, which will serve as a strategy that can be added to the conventional monitoring methods. Therefore, GPS data observations were conducted for two days using two GPS receivers with data sampling intervals of 0.01 seconds- 100 Hz. The Continuous Wavelet Transform (CWT) was used as filtering model to analyze the frequency response of the bridge generated by the residues of the L1 double difference through the highest GPS satellite constellation.

Keywords: - Small Concrete Bridge; GPS; Dynamic Monitoring.

I.

INTRODUCTION

In recent years Brazil has witnessed a significant increase of early deterioration in the special works caused by the lack of preventive maintenance programs for such structures. Albeit Brazil's regulatory bodies provide the complete procedure for inspecting and ensuring the integrity of OAEs, in most cases the pathologies are detected and measures are taken only when the structural deterioration reaches a critical state or when it poses risksto the users. A study conducted by SINAENCO (National Association of Architectural and Consulting Engineering Companies), entitled "Infrastructure of the City: Expiration of theValidity Period" shows the need for a permanent structural conservation and resource managementpolicy. Regarding the special structural works in the city of São Paulo, the study shows that there are 240 bridges and overpasses in state of deterioration regardsinsignificantconservation investments in recent years, corresponding to 0.38% of the final construction cost [18], [1].

This paper presents the partial results of the dynamic monitoring of fluctuations of one of theroadway spans of a small concrete bridge, which used Global Positioning System (GPS). The tests presented below are part of the continuity development of a method, initiated in 2000, in order to improve the GPS detection threshold so that it can also be used in the dynamic monitoring of small rigid-frame bridges.

From the beginning of this work, the Phase Residual Method (PRM) was implemented in the monitoring of medium and large sized suspension and cable bridges [10], [11], [12], [17].

As the Brazilian highway network is almost entirely composed of small and midsized concrete bridges it was necessary to continue the development of the method. The goal is that the ease and speed of using the GPS, as a

tool for monitoring dynamic oscillations of small and medium sized concrete bridges, can be widely used by engineering, also in Brazil.

ThePhase Residual Method, which in its processing uses theL1 carrier-phase data transmitted by only two satellites to the GPS receivers – the mobile one installed at the bridge, and the fixed one installed at the correction station outside the structure. The method includes the use of an electromechanical oscillator with known amplitude and specifically designed to install the GPS antenna, which enables calibrating the degree and frequency of oscillation in the structure. The following results are from initial field tests, carried outon a small reinforced concrete bridge, located at theFernão Dias Highway, in the city ofExtrema, southern Minas Gerais [2].

II. THEORETICAL BASIS OF THE METHOD USED

The method applied in this paper uses data from the GPS positioning system, transmitted by the L1 carrier phase, based on Michelson's interferometer principle, already published in previous works[10], [11], [12],[17].In essence, this methodology consists in the application of the principle of interference on the signals transmitted by the satellites of the GPS constellation to measuredistances by the phase changecaused on one of the signals. In this case, a mobile receiver is installed on the structure to be monitored and the other receiver (fixed or base), responsible for the differential corrections, is installed on a base with known coordinates. Figure 01 illustrates a GPS antenna fixed on the central span of a suspension bridge, excited by dynamic load – in this case, the crossing of a truck. The frequency and oscillation amplitude of the central span are determined from the analysis of the GPS signals collected.



The method used, based on the principle of the interferometer phase, only requires the data collection from two satellites, with phase close to 90° and no more than one constellation with four satellites. Thus, to measure a vertical displacement for example, a satellite should have an altitude of approximately 90° and the other with an altitude close to the horizon (Figure 02). In the processing of dual phase difference, the lower satellite is considered as the reference satellite, which allows obtaining the vector of the residues from the highest satellite, here termed as 'measuringsatellite'.



Figure 2 - Configuration of the reference Satellite and measuring Satellite [2].

1.1 Electro-Mechanical Oscillator

An electromechanical oscillator (Electromechanical Oscillator – EMO) was developed to calibrate the measurement of previously unknown dynamic displacements, which applies controlled movements, related to

amplitude and speed of the structure in the GPS antenna which sustains the movements of the structure. The oscillator is powered by a 9V battery that helps maintain a constant oscillation in the frequency range measured. Figure 3 shows the JAVAD GPS antenna positioned on the oscillator, which is installed on a geodetic pillar of IBGE, during the coordinated transport to the nearby region of the bridge in question.

Figure 3 -Detail of the Electromechanical Oscillator with the JAVAD antenna and oscillator



1.2 Spectral Data Analysis

For the spectral analysis of residues from the GPS processing of the dual-difference phase, we used the Continuous Wavelet Transform – CWT), using the Morlet mother wavelet [15].

CWT allows analyzing spectral variations with different time-frequency-space resolutions. This method is also recommended for specific applications such as noise removing methods, since it uses mathematical reasoning in the classical methods developed byJoseph Fourier (The well knownFourierTransform). This means that a signal can be mapped in a time-scale plane through a scalegram[7],[16], [18].

CWT has a setof "linear" operations (convolution), which can be used in non-stationary signal studies for extracting frequency variation information and to detect oscillations of the structures located in the time scale with its spatial location. This method has been widely used in various fields of research and applications, such asin geophysical, hydrology, climate data and medical analysis, in sound studies, GPS data analysis, and other areas, see[4].

The decision to use the CWT with the Morlet mother wavelet was strategic, since the analyzed signals are not represented simultaneously in time and frequency by the classical Fourier transform, as they are non-stationary, and also because of the energy difference. Additionally, these signals have small peak-to-peak amplitude (up to 5 mm) in a low frequency region, detectable structure by using continuous Wavelet Transforms.

To this end, and as mentioned earlier, the Morlet mother wavelet was used, defined as follows [15]

$$\Psi_{o}(\eta) = \pi^{-1/4} e^{iw_{o}\eta} e^{-1/2\eta^{2}}$$
(1)

Where:

 w_o is the dimensionless cutoff frequency (equal to 6.0, see Torrence & Compo, 1998); and η is the dimensionless time.

When this type of filter is used for extracting features from a time series, theMorlet wavelet is a good choice, since it provides excellent balance between time and frequency localization. The CWT conceptregards theband-pass filter applied to the time series.

The CWT of a time series (f(t), t = 1, ..., N) with time intervals uniformly distributed by dt is defined as the convolution f(t) with the Morletmother wavelet, scaled and normalized:

$$W_{j,k}(t) = \frac{1}{\sqrt{j}} \int_{t=1}^{N} f(t) \Psi_0\left(\frac{t-k}{j}\right) dt$$
(2)

Where $W_{j,k}(t)$ is the similarity between the mother wavelet function and the time series f(t), in other words, the higher the value of $W_{j,k}(t)$, the higher the similarity between the function analyzed and the Morlet wavelet function, which modulates the signal analyzed.

For a function to be considered a Mother Wavelet function(WF), represented by Ψ_0 , the following key properties must be satisfied:

1st Property: The integral of the function must be zero, $\int_{-\infty}^{+\infty} W(x) dx = 0$

$$\int_{-\infty}^{+\infty} \Psi_0(t) dt = 0$$
(3)

Equation (3) ensures that the WF assumes a wavelike shape.

This condition is known as an "admissibility" condition. This ensures the existence of the "Inverse Wavelet Transform", in the composition of the original series.

2nd Property: Must haveenergy unit, i.e.,

$$\int_{-\infty}^{+\infty} |\Psi_0(t)|^2 = 1$$
 (4)

Equation (4) ensures that the WF has compact support, that is, a rapid e-folding time, which ensures that the motherwavelet presents spatial location, a large differential relative to Classical Fourier Transform.

III. CONCRETE BRIDGE OVER THE JAGUARIRIVER, MINAS GERAIS

The concrete bridge over the Jaguari River (Figure 4) was chosen for the experiments because it is a small concrete overpass.



The bridge is situated over the Jaguari River at a stretch of the Fernão Dias highway - BR 381, km 947, of the southbound access of Extrema in southern Minas Gerais, near the border with the state of São Paulo (Figure 5).



Figure 5 – An overview of the Fernão Dias Highway in Brazil.

The study focused only on the southern bridge due to its higher vibration and higher vertical displacement on account of the local traffic load. The structural behavior has been consistently observed using structural and geotechnical methods by the research team coordinated by Professor Túlio Nogueira Bittencourt, from theLaboratory of Structures and Structural Materials of theDepartment of Structures and Foundations at the Polytechnic School of the University of São Paulo.

The structure of the south bridge is divided into five symmetrical spans of: 20m, 26m, 30m, 26m and 20m, with the superstructure supported by 6 pairs of pillars. It is continuous and has no expansion joints, which shows at the ends an abutment with the reinforced concrete transition slab, continuing on the road. Its longitudinal axis has aslope of 5.9% (southward), and continuous roadway superelevation of 8% and projection curvature radius of 305.50 m. Due to the superelevation there is a discrepancy between the pairs of pillars of approximately 50 cm. The total width of the roadway is 11.70 m, of which 10.90 m is divided by the two traffic lanes and a shoulder lane, plus 0.40m for each New-Jersey barriersmade of reinforced concrete. The cross section is the π spaced at 6.40 m between the stringers (center to center); of 40 cm in width and a total height of 2.80 m [1]

1.3 Numerical Modeling and Load Test

The numerical modeling of the bridge over the Jaguari River was based on the Finite Element Method. Five elastic-linear models were studied and created and one non-linear model using the SAP2000® V14 and Midas Fx+ for DIANA® softwares [1]

The dynamic analysis was performed in order to obtain parameters that canevaluate the structural behavior under free vibration. The spectral content analyzed enabled to verify themost excited frequencies, which may correspond to the first symmetric bending frequencies in the longitudinal vertical plane of the bridge. These frequencies are in the range of 5 Hz to 10 Hz.

Figure 6 shows the identification and location of the sensors and transducers used in the monitoring campaign carried out in 2011. The S1 session data were compared with the GPS collected data for exhibiting the largest vertical displacement of 3.55 mm [1]

Figure 6 –Instrumentation layout of the 30 m bridge span over the Jaguari river [1].



Table 1 shows the expected vertical displacement for the LB stringer from the linear analysis results.

Table1 –Linear Analysis: Comparative vertical displacement of LV Stringer, 30 m span (Andrade *et al.*, 2013).

Modulus Displacement (mm) Relative error	(Andrade <i>et al.</i> , 2015).				
	Modulus ofElasticity (GPa)	Displacement (mm)		- Relative error	
(GPa) (GPa) (Model (NM) Test Vehicle (TV) (%)		Numeric Model (NM)	Test Vehicle (TV)	(%)	
24.68 2.933 2.55 17.37	24.68	2.933	3.55	17.37	
23.80 3.047 5.55 14.16	23.80	3.047		14.16	

The monitoring stage with controlled traffic was conducted on October 19, 2011. To perform this stage, the traffic was completely stopped, only allowing the passage of the test vehicle.

The test vehicle consisted of a five-axle truck, 2 axles on the mechanical horse and 3 axles on the semi-trailer, as shown in Figure 7. The total weight of the loaded and unloaded test vehicle was 44.3 t and 16.7 t, respectively [1]

Figure 7 – The test load vehicle used in the test over the bridge [1].



Figure 8 illustrates the acceleration and frequency in the vertical direction for monitoring the normal traffic, on October 10, 2011.

Figure 8 –Acceleration spectrum (frequency) in the vertical direction for monitoring normal traffic on 10/10/2011 [1].



IV. DATA COLLECTION WITH 100HZ GPS RECEIVER

Analyses to test the GPS as a tool for monitoring the central span of the concrete bridge was conducted in three stages: the first stage consisted of the coordinated transportation to the region close to the bridge; the second one used only the EMO with controlled amplitude and frequency, on a geodetic framework for tests to detect vertical deflections of up to 5 mm. In the third and most complete stage, observations were made on the reinforced concrete bridge inExtrema, Minas Gerais.

All data collection stages used two GPS receivers (JAVAD, SIGMA model), with data logging of 100 Hz and chokering antennas (RegAnt_DD JAVADmodel).

1.4 Coordinate Transportation for the Region Nearest to the Bridge

The procedure to characterize the behavior of the bridge with the GPS receiver was done by static relative positioning, thus a geodetic mark with adjusted coordinates was used.

There was no geodetic framework in the vicinity of the bridge that could be used for post-processing the observations. This situation then required building a geodetic mark (Figure 9a) near the bridge which would meet the standards and quality standards suggested by the IBGE [2]

Figure 9 – (a) Construction coordinated by the author who is from IFSULDEMINAS (b) The construction project elaborated by IBGE



The construction site of the geodesic landmark was the center of a roundabout of kilometer 947 south, of the Fernão Dias highway, 300 m from the bridge under observation. Figure 9 illustrates the design situation of thenew framework implemented, the roundabout next to the bridge, the bridge and the base point for the coordinate transport, situated in IFSULDEMINAS - Inconfidentes Campus.

Thus, a cylindrical concrete pillar was built (30 cm diameter and 1.20 m in height). The framework basehas a rectangular shape of 1.40 mx 50 cm across and 60 cm in depth. The structure comprisediron stakes (3/8 inch iron) with the same length of the drilled holes, and withbinding points inserted in the holes. A radier of 1.30 m 40 cm was also placed (Figure 9b).

After the construction of the frame, the GPS datacollection was performed (Figure 10) for the coordinated transport. This procedure was performed on 3 different days, with 6 tracking hours for its approval by the qualified institution.

The engineers of IBGE'sGeodesy Department performed the proper processing and adjustments of the coordinates with the Bernese scientific software version 5.0, and its approval, including it in the Brazilian geodetic network in June 2013.



Figure 10 – Data collection in the GeodeticPillar for its approval by IBGE

1.5 EMO Simulations

The first test used the OEM device (motion simulator) and the JAVAD SIGMA receiver installed on it, which was placed on the geodesic mark located atFederal Sul Institute of Minas Gerais - Inconfidentes Campus, Minas Gerais, Brazil, according toFigure 11.

TheOEMwas used to apply a controlled vertical movement to the GPS antenna with a frequency and amplitude of 0.4 Hz and 3.8 mm, respectively.To record the GPS constellation observations, theJAVAD SIGMAreceivers were used with a recording rate of 100 Hz, for 3 minutes.The result of this test was decisive for the test in the actual structure, in this case, the concrete bridge over the Jaguari River.

Figure 11 – GPS antenna operating on the Electromechanical Oscillator and Pillar of IBGE in IFSULDEMINAS – Inconfidentes Campus.



1.6 EMO simulation results

Figure 12 (below) shows the Energy Spectrumof the residues from the GPS processing of the phase double difference, obtained with the Morlet mother wavelet. The mathematical model used to generate the spectrum was set to show phenomena with 5% statistical significance, that is, 95% ofconfidence –whose significance level is limited by the contour represented by a black line inside the Cone of Influence (COI, [3], [4]). The information from the frequency spectra,based on the residues due to the motion assigned to EMO, will be used as reference for the other analyses presented in this paper. The axis of abscissa (X-axis) is the number of observations (every 0.01 s), the y-axis on the left is the frequency value in Hertz and the y-axis on the right is the energy intensity scale with which the frequency is displayed in the Cone of Influence area. Also, in this figure, the upper graph refers to the gross residues without any treatment or filter, only normalized byN-1, where N is the number of observations derived from the L1 double difference carrier phase, due to the periodic motion of the EMO. The other energy graphics generated by the mathematical model in Mathlab language follow the same description pattern.

Also inFigure 12, there is a highly significant continuous energy band, detected and represented in red over the entire time series period, which corresponds to the oscillations programmed and applied by EMO in the mobile antenna at approximately 0.4 Hz. Between the 3000 and 4000 observations (Note1), energy records of around 2 Hz are observed –which need to be further investigated with field traffic observations.

Figure 12 – Energy Spectrum (bottom, left) of the residues of the time series for the3000-6000observations, with a significance level of 5% limited by theCOI.



V. GPS DATA COLLECTION ON THE CONCRETE BRIDGE OVER THE JAGUARI RIVER

The first tests on the bridge were carried out at different times from the conventional load tests thusfar been conducted by the researchers from the group of Professor Túlio Nogueira Bittencourt. This was due to the need for a coordinated transportation –construction of the pillar at the roundabout – installation of the pins to fix the GPS antenna on the guardrail of the bridge and tests to confirm the detection threshold of the method. New tests are planned to be conducted simultaneously with the load test, the instrumentation with accelerometer and GPS receivers.

1.7 Instrumentation Layout

The instrumentation and GPS data collection took place with the bridge under local highway traffic.During the data collection period, the bridge was under constant traffic by the different categories of heavy vehicles transiting the highway in question. Figures 13a, 13b and 13c show a more detailed lower view of the structure and over the monitored bridge, with the mobile GPS antenna installed on the guardrail, using epoxy to fix the pin.

Figure 13 – (a) bottom view of the bridge, (b) view of the bridge during the tracking with the GPS receiver, (c) detail of pin fixed with epoxy.



Figure 14 illustrates the location of the mobile GPS receiver (rover) on the center of the largest central span of the bridge. The electromechanical oscillator was adjusted to apply a vertical offset of 3.8 mm and frequency of 0.40 Hz. There were two-measurement phaseon the bridge, one using EMO and the other one without the device.

Figure 13 –Sketch of the bridge with the GPS receiver positioned on the30m span.



1.8 GPS Data Collection

At this monitoring stage the GPS antenna was installed on the Jaguari Bridgethrough a forced centering universal threaded pin fixed to the middle and edge of the bridge, as shown in Figure 13b. The GPS base was installed on another geodesic landmark built for this purpose (Figures 09 and 10), 300 m from the bridge under test. The data collection rate of the receptors was 100 observations per second.

The Justin JAVAD v. 2.107software was used to process the data collected over the bridge. According to the methodology, satellite 11 was used as satellite reference because it was on the horizonwith11° of altitude, and satellite 14 was chosen as "measuring"satellitebecause it is located closer to the zenith, with 87° of altitude,illustrated in the SKYPLOT ofFigure 14.

Figure 14 - SKYPLOT of GPS Constellation in the Bridge Experiment on 17/06/2013



1.9 Analysis of Results

Figure 15 shows the first 20 seconds of data collected with a series of 2000observations, with intervals of 0.01 s. The first graph illustrates the residues of double phase difference between satellites 11 and 14. A high energy level is observed due to the multipath of the GPS signal over the frequency range close to the 0.05 Hz value [9] – value for the GPS JAVAD SIGMA receivers with multipath attenuation. Additionally, high energy values with 4 to 8 Hz frequency were observed, corresponding to the frequency band detected during the load test, with the accelerometer installed in the stringer of the 30 m span, under the action of normal traffic. Thus, at this research stage the vertical movements of 30 m span with displacement amplitude of 3.55 mm were identified.



Figure 15 –CWT of the residues of the time series of 0-2000observations, with a significance level of 5% limited by the COI.

Highenergy level was also observed in the region with the 600 and 1800periods. At this stage it is assumed that it may be the response of this span to the passage of larger vehicles such as road trains. In the next research stage there is the comparison of the GPS data recorded period-to-period for the filmed vehicles that passed through the bridge or the data of the motion weighing sensor that is being installed at the nearby toll plaza.

Figure 16 shows the data collected in the following 20 seconds (ofFigure 15). Within the cone of influence, three regions areagain observed, determined by A, B and C, with a high level of energy due to the passage of heavier vehicles such as single or road trains, as mentioned above and to be elucidated. On the other hand, the frequencies in the 4 to 8 Hz range are identified, coinciding with the numerical model and with the load test values performed in 2011. A priori, these frequencies show the possibility of energy transfer between scales, in other words, between the natural vibration of the bridge, with the frequency generated by the passage of vehicles. However, a control experiment isviable for this assertion.

Figure 16-CWT of the residues of the time series of 2000-4000 observations, with a significance level of 5%



VI. CONCLUDING REMARKS

The initial results from the method application that combines Global Positioning System (GPS), Phase Residue Method (PRM) and CWT as a civil engineering tool to monitor the vibrations of small structures allowed to detect vertical deflection frequencies with peak-to-peak amplitude of 3.55 mm, with a high confidence level (95%). The next steps consist of the testsconducted on the same bridge, with joint GPS technology and accelerometers.

The initial tests performed at IFSULDEMINAS were essential for planning the other monitoring projects – as the results achieved were consistent with the expected quality for a reference. The CWT images of the preliminary tests show that the resonance frequency stability of the wave signal justifies the quality of the combination and methodology used. The data were filtered and presented at high energy concentration frequency in most of the time series, around 0.4 Hz, exactly in the order in which the oscillation frequency was set.

The results of the Jaguari Bridge also served to broaden the investigation from the perspective of defining the spectral behavior that each type of road vehicle causes on the bridge. Thus, from the GPS data analysis and the CWT results, it is possible to provide a response spectrum of the span for each type of vehicle.

According to the library, only GPS data needs to be collected, without having to interrupt traffic on concrete bridges with similar structural configurations.

Based on these results, it is concluded that the proposed combination of usinghigh-data rate (100 Hz) GPS receivers with CWT filtering techniques allows to detect, with confidence level of 95%, the senoidal waves from the vertical deflections of the 30 m span length concrete highway bridge.

The next research stages intend toperform traffic analysis and create a response library of the 30 m span for the several types of vehicles crossing, by wavelet analysis, and detail the road movement during the crossing of vehicles.

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